Online Appendix:
Oil and macroeconomic (in)stability

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Abstract

This appendix contains supplementary material not included in the paper. Section 1 describe the data, Section 2 gives details on the New-Keynesian model, Section 3 gives details on the VAR model for oil and the rest of the world, Section 4 specifies the different regimes and 5 gives some additional results.

1 Data and transformations

The model is is estimated using quarterly data from the period 1965Q1–2014Q1. We have 8 observables in the system. We list all the observables together with the variable name used in the model and the corresponding equations (measurement equations) in Appendix 2.1: the federal funds rate (r, Eq. 28), world GDP growth (ΔGDP_t^W, Eq. 29), GDP growth (ΔGDP, Eq. 30), investment growth (ΔINV, Eq. 31), consumption growth ($\Delta CONS$, Eq. 32), wage inflation ($\Delta WAGES$, Eq. 33), CPI-based inflation (ΔCPI, Eq. 34) and oil price inflation (ΔP^{OIL}, Eq. 35).

All the series with the exception of the growth rate of world activity were downloaded from the FRED database.[1] We calculate real per capita values for GDP, consumption and investment. For world activity we use quarterly GDP growth (percentage change) for the OECD countries. The series is named *OECD - total* and is downloaded from OECD.[2]

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[1] See `http://research.stlouisfed.org/fred2/`. In the FRED database, the nominal GDP series is denoted *gdp*, the GDP deflator is named *gdpdef*, the federal funds rate is named *fedfunds*, the WTI series is named *dcoilwtico*, investments is named *gpdi*, wages is named *pcec*, consumption is named *pcec*, the labor force is named *clf16ov* and the CPI is named *cpiaucsl*.

[2] `www.data.oecd.org`

2 Model derivations

The household problem

Households maximize utility subject to a budget constraint and the law of motion for capital. The Lagrangian for the household problem is given by

$$\begin{aligned}
\mathcal{L}_{HH} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ z_t \left[\frac{\left(\frac{C_t - \chi C_{t-1}}{A_t^C}\right)^{1-\sigma}}{1-\sigma} - \kappa_t \frac{n_t^{1+\vartheta}}{1+\vartheta} \right] \\
&- \Lambda_t \left(P_t C_t + P_t I_{K,t} + D_{t-1} r_{t-1} + P_t T_t + F_t - W_t n_t - R_{K,t} K_{t-1} - D_t - DIV_t \right) \\
&- \Lambda_t Q_{K,t} \left(K_t - (1-\delta) K_{t-1} - \left[1 - \frac{\phi_k}{2} \left(\frac{I_{K,t}}{I_{K,t-1}} - \exp(g_{ik}) \right)^2 \right] I_{K,t} A_t^{IK} \right) \Bigg\},
\end{aligned}$$

where $\beta \in (0,1)$ is the subjective discount factor, $\sigma > 0$ is the intertemporal elasticity of substitution, ϑ is the inverse of the Frish elasticity, $\delta \in (0,1)$ is the depreciation rate of capital, ϕ_k governs the degree of investment adjustment costs, and g_{ik} is the growth rate of investment. The first order conditions:

$$\begin{aligned}
C_t &: \; z_t (C_t - \chi C_{t-1})^{-\sigma} (A_t^C)^{\sigma-1} = \Lambda_t P_t \\
D_t &: \; \Lambda_t = \beta \mathbb{E}_t \big[\Lambda_{t+1} r_t \big] \\
K_t &: \; \Lambda_t Q_{K,t} = \mathbb{E}_t \Big[\beta \Lambda_{t+1} \big(R_{K,t+1} + Q_{K,t+1}(1-\delta) \big) \Big] \\
I_{K,t} &: \; \Lambda_t P_t = \Lambda_t Q_{K,t} A_t^{IK} \left[1 - \frac{\phi_k}{2} \left(\frac{I_{K,t}}{I_{K,t-1}} - \exp(g_{ik}) \right)^2 - \phi_k \left(\frac{I_{K,t}}{I_{K,t-1}} - \exp(g_{ik}) \right) \frac{I_{K,t}}{I_{K,t-1}} \right] \\
&+ \mathbb{E}_t \left[\phi_k \beta \Lambda_{t+1} Q_{K,t+1} A_{t+1}^{IK} \left(\frac{I_{K,t+1}}{I_{K,t}} - \exp(g_{ik}) \right) \left(\frac{I_{K,t+1}}{I_{K,t}} \right)^2 \right].
\end{aligned}$$

The first order condition for C_t gives Equation 6, K_t gives Equation 9, and $I_{K,t}$ gives Equation 10. We also define the stochastic discount factor as

$$m_t \equiv \mathbb{E}_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \right\},$$

and together with the first order condition w.r.t. D_t, we get Equation 7 and 8.

Optimal wage setting a la Rotemberg

Wages are chosen by minimizing household disutility of working subject to the budget constraint and a quadratic cost of adjusting wages. Wage inflation is defined as

$$\pi_t^w = \frac{W_t}{W_{t-1}}.$$

We assume wage indexation given by

$$\ddot{\pi}_t^w \equiv (\pi_{t-1}^w)^{\gamma_w} (\bar{\pi}^w)^{1-\gamma_w}.$$

The Lagrangian for the labor unions:

$$
\begin{aligned}
\mathcal{L}_{WS} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ z_t \kappa_t \left[- \frac{\left(\left(\frac{W_{j,t}}{W_t} \right)^{-\upsilon} n_t \right)^{1+\vartheta}}{1+\vartheta} \right] \\
&- \zeta_t \Big[P_t C_t + P_t I_{K,t} - D_t - W_{j,t} \left(\frac{W_{j,t}}{W_t} \right)^{-\upsilon} n_t - R_{K,t} K_{t-1} + D_{t-1} r_{t-1} \\
&+ P_t TAX_t + \frac{\xi}{2} \left[\frac{W_{j,t}}{W_{j,t-1}} - \ddot{\pi}_t^w \right]^2 W_t n_t - DIV_t \Big] \Big\},
\end{aligned}
$$

where υ is the elasticity of substitution between different types of labor, and ξ governs the degree of cost of changing wages. First order conditions w.r.t. $W_{j,t}$:

$$
\begin{aligned}
0 &= \upsilon z_t \kappa_t \frac{n_t}{W_t} \left(\left(\frac{W_{j,t}}{W_t} \right)^{-\upsilon} n_t \right)^{\vartheta} \left(\frac{W_{j,t}}{W_t} \right)^{-\upsilon-1} + \zeta_t (1-\upsilon) W_{j,t}^{-\upsilon} W_t^{\upsilon} n_t \\
&\quad -\Lambda_t \xi \left[\frac{W_{j,t}}{W_{j,t-1}} - \ddot{\pi}_t^w \right] \frac{W_t n_t}{W_{j,t-1}} + \mathbb{E}_t \left\{ \beta \Lambda_{t+1} \xi W_{t+1} n_{t+1} \frac{W_{j,t+1}}{W_{j,t}^2} \left[\frac{W_{j,t+1}}{W_{j,t}} - \ddot{\pi}_t^w \right] \right\} \\
&= \upsilon z_t \kappa_t \frac{n_{j,t}^{1+\vartheta}}{W_{j,t}} + \Lambda_t (1-\upsilon) n_{j,t} \\
&\quad -\Lambda_t \xi \frac{W_t n_t}{W_{j,t-1}} \left[\frac{W_{j,t}}{W_{j,t-1}} - \ddot{\pi}_t^w \right] + \mathbb{E}_t \left\{ \beta \Lambda_{t+1} \xi W_{t+1} n_{t+1} \frac{W_{j,t+1}}{W_{j,t}^2} \left[\frac{W_{j,t+1}}{W_{j,t}} - \ddot{\pi}_t^w \right] \right\}.
\end{aligned}
$$

Invoking symmetry and dividing throughout by $\Lambda_t(\upsilon - 1) n_t$:

$$
\begin{aligned}
0 &= \frac{\upsilon}{\upsilon - 1} z_t \kappa_t \frac{n_t^{\vartheta}}{\Lambda_t W_t} - 1 \\
&\quad - \frac{\xi}{\upsilon - 1} \frac{W_t}{W_{t-1}} \left[\frac{W_t}{W_{t-1}} - \ddot{\pi}_t^w \right] + \mathbb{E}_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\xi}{\upsilon - 1} \frac{n_{t+1}}{n_t} \left(\frac{W_{t+1}}{W_t} \right)^2 \left[\frac{W_{t+1}}{W_t} - \ddot{\pi}_t^w \right] \right\},
\end{aligned}
$$

which gives Equation 27.

Intermediate goods firms' problem

Firms have the following technology

$$
Y_{it} = A_t \Big[O_{i,t}^{\varrho} K_{i,t-1}^{1-\varrho} \Big]^{\alpha} n_{i,t}^{1-\alpha},
$$

and maximize profits given by

$$
DIV_{i,t} = P_{i,t} Y_{i,t} - W_{i,t} n_{i,t} - R_{K,t} K_{i,t-1} - P_{O,t} O_{i,t} + F_{i,t}.
$$

The firm solves

$$
\max_{n_{i,t}, K_{i,t-1}, O_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} P_{i,t} \left[A_t \Big[O_{i,t}^{\varrho} K_{i,t-1}^{1-\varrho} \Big]^{\alpha} n_{i,t}^{1-\alpha} \right] - W_{i,t} n_{i,t} - R_{K,t} K_{i,t-1} - P_{o,t} O_{i,t} + F_{i,t},
$$

where α, is the share of the oil and capital mix in production and ϱ governs the mix of oil and capital. First order condition w.r.t. $P_{j,t}$:

$$\begin{aligned} O_{i,t}: & \quad \alpha\varrho\frac{\Psi_{i,t}Y_{i,t}}{O_{i,t}} = P_{o,t} \\ n_{i,t}: & \quad (1-\alpha)\frac{\Psi_{i,t}Y_{i,t}}{n_{i,t}} = W_{i,t} \\ K_{i,t-1}: & \quad \alpha(1-\varrho)\frac{\Psi_{i,t}Y_{i,t}}{K_{i,t-1}} = R_{K,t}, \end{aligned}$$

where $\Psi_{i,t}$ is the inverse of the Lagrange multiplier, and can be interpreted as the firms' marginal cost. If we invoke symmetry, the first order condition for $O_{i,t}$ gives Equation 17, $n_{i,t}$ gives Equation 19, and $K_{i,t-1}$ gives Equation 18.

Optimal price setting a la Rotemberg

Firms set prices to maximize revenue, $P_{i,t}Y_{i,t}$, minus cost of producing, $\Psi_t Y_{i,t}$, minus the cost of adjusting the price, $\frac{\omega}{2}P_tY_t\left(\frac{P_{i,t}}{P_{i,t-1}} - \ddot{\pi}_t\right)^2$. Inflation is given by

$$\pi_t = \frac{P_t}{P_{t-1}},$$

and we have price indexation to the previous period

$$\ddot{\pi}_t \equiv \pi_{t-1}^{\gamma_\pi}\bar{\pi}^{1-\gamma_\pi}.$$

We also include a shock to the value of production, called a stochastic subsidy shock $\varepsilon_{\pi,t}$. The firm problem is given by:

$$\max_{P_{i,t}} \mathbb{E}_t \sum_{t=0}^{\infty} m_t \left[P_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t \exp(\sigma_\pi\varepsilon_{\pi,t}) - \Psi_t\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t - \frac{\omega}{2}P_tY_t\left(\frac{P_{i,t}}{P_{i,t-1}} - \ddot{\pi}_t\right)^2\right],$$

where ϵ is the elasticity of substitution between different varieties of goods, and ω governs the cost of adjusting prices. First order condition:

$$\begin{aligned} P_{i,t}: \quad & 0 = (1-\epsilon)Y_{i,t}\exp(\sigma_\pi\epsilon_{\pi,t}) + \epsilon\Psi_t\frac{Y_t}{P_{i,t}} - \omega\frac{P_t}{P_{i,t-1}}Y_t\left[\frac{P_{i,t}}{P_{i,t-1}} - \ddot{\pi}_t\right] \\ & + \mathbb{E}_t\left\{\omega m_{t+1}P_{t+1}Y_{t+1}\frac{P_{i,t+1}}{P_{i,t}^2}\left[\frac{P_{i,t+1}}{P_{i,t}} - \ddot{\pi}_{t+1}\right]\right\}. \end{aligned}$$

Invoking symmetry and dividing throughout by $(\epsilon-1)Y_t$:

$$\begin{aligned} 0 = & -\exp(\sigma_\pi\varepsilon_{\pi,t}) + \frac{\epsilon}{\epsilon-1}\frac{\Psi_t}{P_t} - \frac{\omega}{\epsilon-1}\frac{P_t}{P_{t-1}}\left[\frac{P_t}{P_{t-1}} - \ddot{\pi}_t\right] \\ & + \mathbb{E}_t\left\{\frac{\omega}{\epsilon-1}m_{t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}}{P_t}\right)\left[\frac{P_{t+1}}{P_t} - \ddot{\pi}_{t+1}\right]\right\}, \end{aligned}$$

which gives Equation 20.

Aggregation and market clearing

The market for goods clear,

$$Y_t = C_t + I_{K,t} + G_t.$$

Firm profits is given by

$$DIV_t = P_t Y_t - W_t n_t - R_{K,t} K_{t-1} - P_{o,t} O_t + F_t,$$

which gives Equation 25.

2.1 The stationary system

To get a stationary system we use the following variable transformations:

$c_t \equiv \frac{C_t}{A_t^C}$, $\lambda_t \equiv A_t^C \Lambda_t P_t$, $\mu_{a,t} \equiv \frac{A_t}{A_{t-1}}$, $\mu_{ac,t} \equiv \frac{A_t^C}{A_{t-1}^C}$, $\mu_{aik,t} \equiv \frac{A_t^{IK}}{A_{t-1}^{IK}}$, $i_{k,t} \equiv \frac{I_{K,t}}{A_t^C}$, $f_t \equiv \frac{F_t}{A_t^C P_t}$, $g_t = \frac{G_t}{A_t^C}$, $tax_t \equiv \frac{TAX_t}{A_t^C}$, $w_t = \frac{W_t}{A_t^C P_t}$, $k_t \equiv \frac{K_t}{A_t^C A_t^{IK}}$, $r_{k,t} \equiv \frac{R_{K,t} A_t^{IK}}{P_t}$, $d_t \equiv \frac{D_t}{A_t^C P_t}$, $div_t \equiv \frac{DIV_t}{A_t^C P_t}$, $q_{k,t} \equiv \frac{A_t^{IK} Q_t^K}{P_t}$, $y_t \equiv \frac{Y_t}{A_t^C}$, $\psi_t \equiv \frac{\Psi_t}{P_t}$, $o_t \equiv \frac{O_t}{A_t^C A_t^{IK}}$, $p_{o,t} \equiv \frac{P_{o,t} A_t^{IK}}{P_t}$
With these definitions the stationary system, corresponding to the set of equations coded up in the RISE toolbox, is as follows:
The labor preference shock process is given by

$$\kappa_t = \kappa_{t-1}^{\rho_\kappa} \kappa^{1-\rho_\kappa} \exp(\sigma_\kappa \varepsilon_{\kappa,t}). \tag{1}$$

The intertemporal preference shock process is given by

$$\log(z_t) = \rho_z \log(z_{t-1}) + \sigma_z \varepsilon_{z,t}. \tag{2}$$

The households have the following budget constraint

$$c_t + i_{k,t} + \frac{d_{t-1} r_{t-1}}{\mu_{ac,t} \pi_t} + tax_t + f_t = w_t n_t + \frac{r_{k,t} k_{t-1}}{\mu_{aik,t} \mu_{ac,t}} + d_t + div_t. \tag{3}$$

Capital accumulation with capital adjustment costs

$$k_t = \frac{(1-\delta) k_{t-1}}{\mu_{ac,t} \mu_{aik,t}} + \left(1 - \frac{\phi_k}{2} \left(\frac{\mu_{ac,t} i_{k,t}}{i_{k,t-1}} - \exp(g_{ik})\right)^2 i_{k,t}\right). \tag{4}$$

The capital investment shock process

$$\mu_{aik,t} = \exp(g_{aik} + \sigma_{aik} \varepsilon_{aik,t}). \tag{5}$$

Household FOCs give optimal allocation of consumption between periods

$$z_t \left(c_t - \frac{\chi c_{t-1}}{\mu_{ac,t}}\right)^{-\sigma} = \lambda_t. \tag{6}$$

The pricing kernel is determined by the growth in marginal utility, and the stochastic discount factor is given by,

$$m_t = \mathbb{E}_t \left[\frac{\beta \lambda_{t+1}}{\lambda_t \mu_{ac,t+1} \pi_{t+1}}\right]. \tag{7}$$

The stochastic discount factor is equal to the inverse of the interest rate

$$m_t = \frac{1}{r_t}. \tag{8}$$

Optimal capital allocation is governed by

$$\lambda_t q_{k,t} = \mathbb{E}_t \left\{ \frac{\beta \lambda_{t+1}}{\mu_{ac,t+1}\mu_{aik,t+1}} \left(r_{k,t+1} + q_{k,t+1}(1-\delta) \right) \right\}. \tag{9}$$

The optimal level of investment is governed by

$$\begin{aligned} \lambda_t &= \lambda_t q_{k,t} \left(1 - \frac{\phi_k}{2} \left(\frac{\mu_{ac,t} i_{k,t}}{i_{k,t-1}} - \exp(g_{ik}) \right)^2 \right) \\ &- \lambda_t q_{k,t} \phi_k \left(\frac{\mu_{ac,t} i_{k,t}}{i_{k,t-1}} - \exp(g_{ik}) \right) \left(\frac{\mu_{ac,t} i_{k,t}}{i_{k,t-1}} \right) \\ &+ \mathbb{E}_t \left\{ \phi_k \beta \lambda_{t+1} q_{k,t+1} \left(\frac{\mu_{ac,t+1} i_{k,t+1}}{i_{k,t}} - \exp(g_{ik}) \right) \left(\frac{\mu_{ac,t+1} i_{k,t+1}}{i_{k,t}} \right)^2 \right\} \end{aligned} \tag{10}$$

Membership fee to labor unions

$$f_t = \frac{\xi}{2} w_t n_t (\pi_t^w - \ddot{\pi}_t^w)^2. \tag{11}$$

Wage inflation is given by

$$\pi_{w,t} = \mu_{ac,t} \pi_t \frac{w_t}{w_{t-1}}. \tag{12}$$

We have some wage indexation to the previous period, and the aggregate wage index is given by

$$\ddot{\pi}_t^w \equiv (\pi_{t-1}^w)^{\gamma_w} (\bar{\pi}^w)^{1-\gamma_w}. \tag{13}$$

We have some price indexation to the previous period, and the aggregate price index is given by

$$\ddot{\pi}_t \equiv \pi_{t-1}^{\gamma_\pi} \bar{\pi}^{1-\gamma_\pi}. \tag{14}$$

The production technology is given by

$$y_t = \left(\frac{k_{t-1}}{\mu_{ac,t}\mu_{aik,t}} \right)^{\alpha(1-\varrho)} o_t^{\alpha\varrho} n_t^{1-\alpha}. \tag{15}$$

The neutral technology process is given by

$$\mu_{a,t} = \exp(g_a + \sigma_a \epsilon_{a,t}). \tag{16}$$

Firms select the quantity of labor, capital, and oil to maximize profits, which gives

$$p_{o,t} = \alpha \varrho \psi_t \frac{y_t}{o_t}, \tag{17}$$

$$r_{k,t} = \alpha(1-\varrho)\mu_{ac,t}\mu_{aik,t}\psi_t \frac{y_t}{k_{t-1}}, \tag{18}$$

$$w_t = (1-\alpha)\psi_t \frac{y_t}{n_t}. \tag{19}$$

Optimal price setting from the Rotemberg model:

$$\begin{aligned} &\frac{\epsilon}{\epsilon-1}\psi_t - \exp(\sigma_\pi \varepsilon_{\pi,t}) - \frac{\omega}{\epsilon-1}\pi_t(\pi_t - \ddot{\pi}_t) \\ &+ \mathbb{E}_t \left\{ \frac{\omega}{\epsilon-1} m_t \left(\mu_{ac,t+1} \frac{y_{t+1}}{y_t} \right) \pi_{t+1}^2 (\pi_{t+1} - \ddot{\pi}_{t+1}) \right\} = 0. \end{aligned} \tag{20}$$

The central bank follows a Taylor rule given by

$$r_t = r_{t-1}^{\rho} \left(\bar{r} \left(\frac{y_t}{\bar{y}} \right)^{\kappa_y} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\kappa_\pi} \right)^{1-\rho} \exp(\sigma_r \varepsilon_{r,t}). \tag{21}$$

Government spending follows an AR(1)

$$g = g_{t-1}^{\rho_g} \bar{g}^{1-\rho_g} \exp(\sigma_g \varepsilon_g). \tag{22}$$

Government spending must be financed by taxes,

$$g_t = tax_t. \tag{23}$$

And aggregate debt is zero

$$d_t = 0. \tag{24}$$

Dividends or profits are given by

$$div_t = y_t - w_t n_t - \frac{r_{k,t} k_{t-1}}{\mu_{aik,t} \mu_{ac,t}} - p_{o,t} o_t + f_t. \tag{25}$$

The growth rate of the consumption process is given by

$$\mu_{ac,t} = \mu_{a,t}^{\frac{1}{1-\alpha}} \mu_{aik,t}^{\frac{\alpha}{1-\alpha}}. \tag{26}$$

Optimal wage setting a la Rotemberg

$$\begin{aligned} &\frac{\upsilon}{\upsilon - 1} z_t \kappa_t \frac{n_t^{\vartheta}}{w_t \lambda_t} - 1 - \frac{\xi}{\upsilon - 1} \pi_t^w (\pi_t^w - \ddot{\pi}_t^w) \\ &+ \mathbb{E}_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t \pi_{t+1} \mu_{ac,t+1}} \frac{\xi}{\upsilon - 1} \frac{n_{t+1}}{n_t} (\pi_{t+1}^w)^2 (\pi_{t+1}^w - \ddot{\pi}_{t+1}^w) \right\} = 0. \end{aligned} \tag{27}$$

2.2 Measurement equations

The observed interest rate is equal to the interest rate in the model

$$r_{obs,t} = r_t. \tag{28}$$

The observed growth rate of world activity is equal to the growth rate of world activity in the model

$$\Delta GDP_{obs,t}^W = \Delta GDP_t^W. \tag{29}$$

The growth rate of output is defined as

$$\Delta GDP_t \equiv \log(\mu_{ac,t}) + \log \left(\frac{y_t}{y_{t-1}} \right). \tag{30}$$

The growth rate of investment is defined as

$$\Delta INV_t \equiv \log(\mu_{ac,t}) + \log \left(\frac{i_{k,t}}{i_{k,t-1}} \right). \tag{31}$$

The growth rate of consumption spending is defined as,

$$\Delta CONS_t \equiv \log(\mu_{ac,t}) + \log \left(\frac{c_t}{c_{t-1}} \right). \tag{32}$$

The growth rate of wages is defined as

$$\Delta WAGES_t \equiv \log(\pi_t^w). \tag{33}$$

The growth rate of the consumer price index is defined as,

$$\Delta CPI_t \equiv \log(\pi_t). \tag{34}$$

The growth rate of oil prices is defined as,

$$\Delta P_t^{Oil} \equiv \log(\mu_{aik,t}) + \log(\pi_t) + \log\left(\frac{p_{o,t}}{p_{o,t-1}}\right). \tag{35}$$

A description of all the endogenous, exogenous, and observable variables is given in the Tables 3, 1 and 2 respectively.

2.3 The steady state

The steady state of the system is given by the following relations

$$n_t = \bar{n}, \quad z_t = 1, \quad \mu_{aik,t} = \exp\left(g_{aik}\right), \quad q_{k,t} = 1$$

$$\mu_{a,t} = \exp\left(g_A\right), \quad \pi_t = \bar{\pi}, \quad d_t = 0, \quad g_{ik} = \log\left(\mu_{ac,t}\right)$$

$$\mu_{ac,t} = \left(\mu_{a,t}\right)^{\frac{1}{1-\alpha}}\left(\mu_{aik,t}\right)^{\frac{\alpha}{1-\alpha}}, \quad p_{o,t} = p_o, \quad r_t = \frac{1}{m_{t,t}}$$

$$m_{t,t} \equiv \frac{\beta}{\mu_{ac,t}\pi_t}, \quad r_{k,t} = \left[\frac{\mu_{ac,t}\mu_{aik,t}}{\beta} - 1 + \delta_K\right] q_{k,t}$$

$$f_t = 0, \quad \pi_t^w \equiv \mu_{ac,t}\pi_t, \quad \ddot{\pi}_t^w \equiv \left(\pi_t^w\right)^{\gamma_w}\left(\bar{\pi}^w\right)^{1-\gamma_w}$$

$$\ddot{\pi}_t \equiv \pi_t^{\gamma_\pi}\bar{\pi}^{1-\gamma_\pi}, \quad \psi_t = \frac{\epsilon - 1}{\epsilon}, \quad \frac{y}{k} = \frac{r_{k,t}}{\alpha\left(1-\varrho\right)\mu_{ac,t}\mu_{aik,t}\psi_t}$$

$$\frac{o}{k} = \frac{\alpha\varrho\psi_t}{p_{o,t}}\frac{y}{k}, \quad k_t = \frac{n_t}{\left[\left(\mu_{ac,t}\mu_{aik,t}\right)^{(1-\varrho)\alpha}\left(\frac{o_t}{k_t}\right)^{-\varrho\alpha}\frac{y_t}{k_t}\right]^{\frac{1}{1-\alpha}}}$$

$$o_t = \left(\frac{o}{k}\right)k_t, \quad y_t = \left(\frac{y}{k}\right)k_t, \quad w_t = \left(1-\alpha\right)\frac{\psi_t}{n_t}y_t$$

$$i_{k,t} = \left[1 - \frac{\left(1-\delta_K\right)}{\mu_{ac,t}\mu_{aik,t}}\right]k_t, \quad g_t = \left(\frac{G}{Y}\right)y_t, \quad tax_t = g_t$$

$$c_t = y_t - tax_t - \left(\frac{r_t}{\mu_{ac,t}\pi_t} - 1\right)d_t - i_{k,t}, \text{ with } c_t > 0$$

$$\lambda_t = z_t\left(c_t - \chi\frac{c_t}{\mu_{ac,t}}\right)^{-\sigma}, \quad div_t = y_t - w_t n_t - \frac{1}{\mu_{aik,t}\mu_{ac,t}}r_{k,t}k_t + f_t$$

$$\kappa_t = \frac{\left(\upsilon - 1\right)w_t\lambda_t}{\upsilon, \quad \kappa = \kappa_t z_t n_t^{\vartheta}}$$

2.4 Description of the variables

In this section we list the variables in our model together with a short description. Table 1 lists the exogenous variables, Table 2 lists the observable variables and Table 3 lists the endogenous variables.

Table 1: Exogenous variables

Variable	Description
ε_A	Neutral technology shock
ε_{AIK}	Investment specific technology shock
ε_G	Government spending shock
ε_κ	Labor preference shock
ε_o	Oil price shock
ε_W	World oil demand shock
ε_π	Stochastic subsidy shock
ε_r	Monetary policy shock
ε_z	Consumption preference shock

Table 2: Observable variables

Variable	Description
$\Delta CONS$	Consumption growth
ΔCPI	CPI inflation
ΔGDP	GDP growth
ΔINV	Investment growth
ΔP^{Oil}	Oil price inflation
$\Delta WAGES$	Wage inflation
ΔGDP^W	World GDP growth
r	Interest rate

Table 3: Endogenous variables

Variable	Description
c	Aggregate consumption
d	Deposits/Loans
$\Delta CONS$	Consumption growth
ΔCPI	CPI inflation
ΔGDP	GDP growth
ΔGOV	Government spending growth
ΔINV	Investment growth
div	Dividends
ΔP^{Oil}	Oil price inflation
$\Delta WAGES$	Wage inflation
f	Membership fees
g	Government spending
i_k	Investment
k	Capital
κ	Labor preferences
λ	Marginal utility of consumption
p_{o}	Real oil price
m	Stochastic discount factor
μ_a	Neutral technology process
μ_{aik}	Capital investment shock process
n	Labor
o	Oil demand
ΔGDP^W	World GDP growth
π	Inflation
$\ddot{\pi}$	Inflation index
π_w	Wage inflation
$\ddot{\pi}^w$	Wage inflation index
p_o	Price of oil
ψ	Real marginal cost
q_k	Price of capital goods
r	Interest rate
r_k	Rental rate of capital
tax	Taxes
w	Wages
y	Output
z	Intertemporal preference shifter

3 A model for oil and the rest of the world

We model growth in world activity and the real oil price jointly in a bi-variate VAR model given by

$$\boldsymbol{A}_0 \begin{bmatrix} \Delta\log(GDP^{\mathrm{W}}_t) \\ \log(p_{o,t}) \end{bmatrix} = \boldsymbol{c} + \sum_{j=1}^{2} \boldsymbol{A}_j \begin{bmatrix} \Delta\log(GDP^{\mathrm{W}}_{t-j}) \\ \log(p_{o,t-j}) \end{bmatrix} + \begin{bmatrix} \sigma^{\mathrm{W}}_t \varepsilon_{W,t} \\ \sigma^{\mathrm{Oil}}_t(\mathcal{S}^{\mathrm{Oil}}_t)\varepsilon_{o,t} \end{bmatrix}, \tag{36}$$

where the $\boldsymbol{A}$ matrices are given by

$$\boldsymbol{A}_0 \equiv \begin{bmatrix} 1 & 0 \\ a_{0,21} & 1 \end{bmatrix}, \ \boldsymbol{A}_1 \equiv \begin{bmatrix} a_{1,11} & a_{1,12} \\ a_{1,21} & a_{1,22} \end{bmatrix} \text{ and } \boldsymbol{A}_2 \equiv \begin{bmatrix} a_{2,11} & a_{2,12} \\ a_{2,21} & a_{2,22} \end{bmatrix}.$$

The impulse responses from the oil – macroeconomic relationship is given in the paper, Table 4 displays the priors and the posterior (modes and medians) for the VAR parameters.

Table 4: Priors and posteriors

	Prior			Posterior			
Param.	Distr.	5%	95%	Mode	Median	5%	95%
$a_{1,11}$	Normal	-1.158	2.132	0.5833	0.5283	0.3927	0.6726
$a_{1,12}$	Normal	-1.644	1.646	0.0019	0.0018	-0.0038	0.0075
$a_{2,11}$	Normal	-1.603	1.686	-0.0027	0.0558	-0.0521	0.1276
$a_{2,12}$	Normal	-1.648	1.642	-0.0030	-0.0030	-0.0069	0.0007
$a_{0,21}$	Normal	-2.079	1.211	-0.3821	-0.4768	-0.7721	-0.2215
$a_{1,21}$	Normal	1.352	4.642	3.073	3.017	2.882	3.1
$a_{1,22}$	Normal	-0.4902	2.799	1.295	1.235	1.167	1.321
$a_{2,21}$	Normal	-4.946	-1.656	-2.938	-3.173	-3.417	-2.923
$a_{2,22}$	Normal	-1.832	1.458	-0.3083	-0.2473	-0.3353	-0.1756

4 Specification of the different regimes

In our model set-up we specify two monetary policy states (hawkish and dovish), two macro volatility states (high and low) and two oil price volatility states (high and low). Combining these states gives us eight possible regimes, see Table 5 for a specification of the different regimes by combining of different states.

Table 5: Specification of the regimes

	Macroeconomic volatility	Oil price volatility	Monetary policy
Regime 1 ($\mathcal{S}_t = 1$)	$\mathcal{S}_t^{Vol}$ = High	$\mathcal{S}_t^{Oil}$ = High	$\mathcal{S}_t^{Pol}$ = Hawkish
Regime 2 ($\mathcal{S}_t = 2$)	$\mathcal{S}_t^{Vol}$ = High	$\mathcal{S}_t^{Oil}$ = High	$\mathcal{S}_t^{Pol}$ = Dovish
Regime 3 ($\mathcal{S}_t = 3$)	$\mathcal{S}_t^{Vol}$ = High	$\mathcal{S}_t^{Oil}$ = Low	$\mathcal{S}_t^{Pol}$ = Hawkish
Regime 4 ($\mathcal{S}_t = 4$)	$\mathcal{S}_t^{Vol}$ = High	$\mathcal{S}_t^{Oil}$ = Low	$\mathcal{S}_t^{Pol}$ = Dovish
Regime 5 ($\mathcal{S}_t = 5$)	$\mathcal{S}_t^{Vol}$ = Low	$\mathcal{S}_t^{Oil}$ = High	$\mathcal{S}_t^{Pol}$ = Hawkish
Regime 6 ($\mathcal{S}_t = 6$)	$\mathcal{S}_t^{Vol}$ = Low	$\mathcal{S}_t^{Oil}$ = High	$\mathcal{S}_t^{Pol}$ = Dovish
Regime 7 ($\mathcal{S}_t = 7$)	$\mathcal{S}_t^{Vol}$ = Low	$\mathcal{S}_t^{Oil}$ = Low	$\mathcal{S}_t^{Pol}$ = Hawkish
Regime 8 ($\mathcal{S}_t = 8$)	$\mathcal{S}_t^{Vol}$ = Low	$\mathcal{S}_t^{Oil}$ = Low	$\mathcal{S}_t^{Pol}$ = Dovish

5 Additional Results

Table 6 – 13 reports the variance decomposition for an oil price shock in the eight different regimes. Figure 1 – 8 displays the smoothed probabilities for being in the different regimes.

Table 9 investigates the robustness of starting the estimation period in 1975Q1 while Table 10 investigates the robustness of ending the estimation period in 2008Q4.

Table 6: Variance decomposition regime 1

	1	4	8	12	16	20	40
GDP growth	0.21	2.30	3.80	3.90	3.90	3.90	4.00
Price Inflation	37.00	35.00	32.00	33.00	34.00	36.00	42.00
Wage inflation	12.00	9.10	8.40	8.90	10.00	11.00	17.00
Interest Rate	10.00	29.00	33.00	34.00	35.00	38.00	48.00

Note: Regime 1: High macro vol & High oil price vol & Hawkish state. Variance Decomposition: Contribution of Oil shocks.

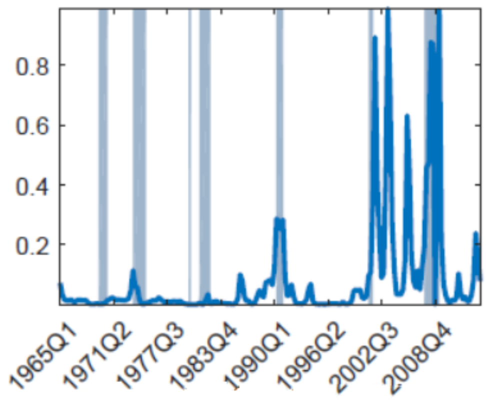

Figure 1: Smoothed probabilities for regime 1

Table 7: Variance decomposition regime 2

	1	4	8	12	16	20	40
GDP growth	4.30	4.40	4.30	4.40	4.40	4.40	4.50
Price Inflation	45.00	42.00	42.00	43.00	45.00	47.00	52.00
Wage inflation	29.00	29.00	31.00	33.00	35.00	36.00	41.00
Interest Rate	13.00	35.00	39.00	41.00	43.00	45.00	52.00

Note: Regime 2: High macro vol & High oil price vol & Dovish state.

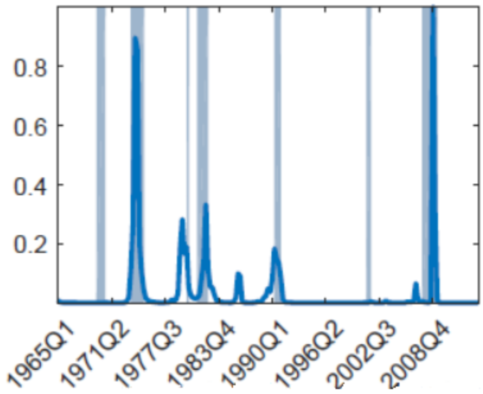

Figure 2: Smoothed probabilities for regime 2

Table 8: Variance decomposition regime 3

	1	4	8	12	16	20	40
GDP growth	0.01	0.18	0.30	0.31	0.31	0.31	0.31
Price Inflation	4.20	3.90	3.50	3.50	3.80	4.00	5.10
Wage inflation	0.99	0.76	0.69	0.73	0.83	0.96	1.60
Interest Rate	0.86	3.00	3.60	3.70	4.00	4.30	6.60

Note: Regime 3: High macro vol & Low oil price vol & Hawkish state.

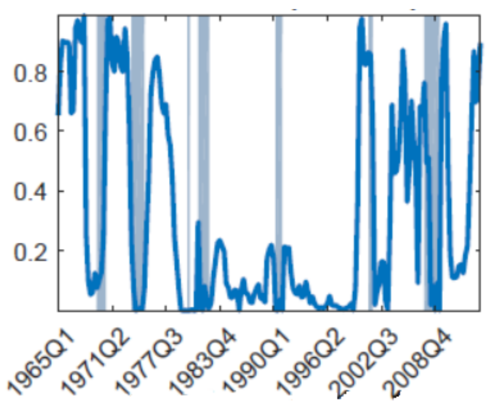

Figure 3: Smoothed probabilities for regime 3

Table 9: Variance decomposition regime 4

	1	4	8	12	16	20	40
GDP growth	0.34	0.34	0.34	0.34	0.35	0.35	0.35
Price Inflation	5.80	5.30	5.10	5.40	5.80	6.20	7.50
Wage inflation	3.00	3.00	3.20	3.50	3.80	4.10	5.00
Interest Rate	1.10	3.80	4.60	4.90	5.40	5.90	7.50

Note: Regime 4: High macro vol & Low oil price vol & Dovish state.

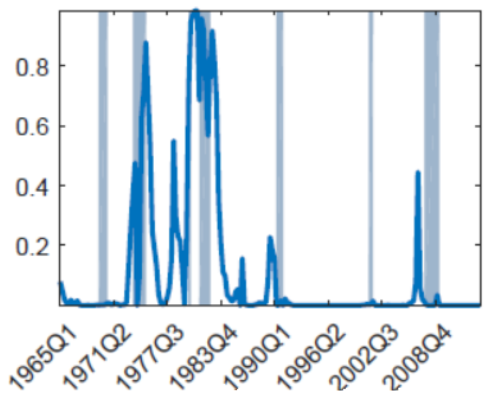

Figure 4: Smoothed probabilities for regime 4

Table 10: Variance decomposition regime 5

	1	4	8	12	16	20	40
GDP growth	0.82	8.50	13.00	14.00	14.00	14.00	14.00
Price Inflation	68.00	65.00	63.00	64.00	66.00	67.00	73.00
Wage inflation	31.00	27.00	26.00	27.00	30.00	33.00	45.00
Interest Rate	12.00	45.00	61.00	65.00	67.00	70.00	79.00

Note: Regime 5: Low macro vol & High oil price vol & Hawkish state.

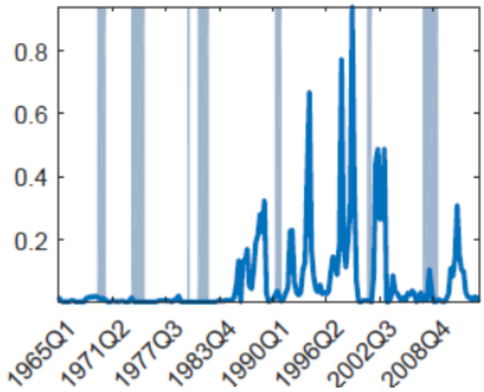

Figure 5: Smoothed probabilities for regime 5

Table 11: Variance decomposition regime 6

	1	4	8	12	16	20	40
GDP growth	16.00	17.00	16.00	17.00	17.00	17.00	17.00
Price Inflation	79.00	78.00	78.00	78.00	80.00	81.00	84.00
Wage inflation	65.00	66.00	67.00	69.00	71.00	73.00	77.00
Interest Rate	15.00	54.00	67.00	71.00	74.00	76.00	80.00

Note: Regime 6: Low macro vol & High oil price vol & Dovish state.

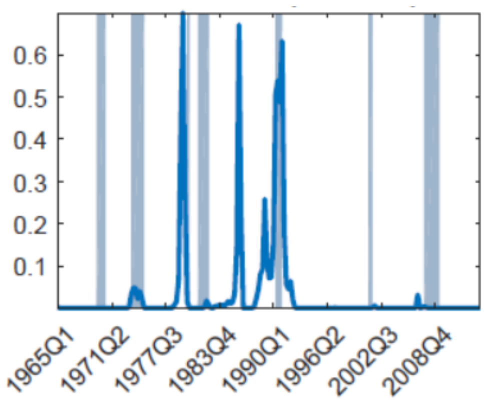

Figure 6: Smoothed probabilities for regime 6

Table 12: Variance decomposition regime 7

	1	4	8	12	16	20	40
GDP growth	0.06	0.70	1.20	1.20	1.20	1.20	1.20
Price Inflation	14.00	12.00	12.00	12.00	13.00	13.00	17.00
Wage inflation	3.30	2.70	2.50	2.70	3.10	3.60	5.80
Interest Rate	1.00	5.90	10.00	12.00	14.00	15.00	22.00

Note: Regime 7: Low macro vol & Low oil price vol & Hawkish state.

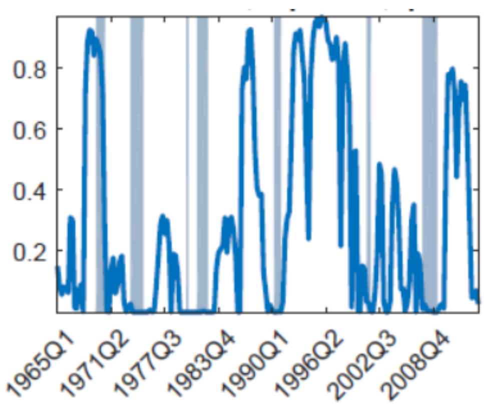

Figure 7: Smoothed probabilities for regime 7

Table 13: Variance decomposition regime 8

	1	4	8	12	16	20	40
GDP growth	1.50	1.50	1.50	1.50	1.50	1.50	1.50
Price Inflation	22.00	21.00	21.00	22.00	23.00	24.00	28.00
Wage inflation	12.00	13.00	13.00	15.00	16.00	17.00	20.00
Interest Rate	1.30	8.00	13.00	16.00	17.00	19.00	23.00

Note: Regime 8: Low macro vol & Low oil price vol & Dovish state.

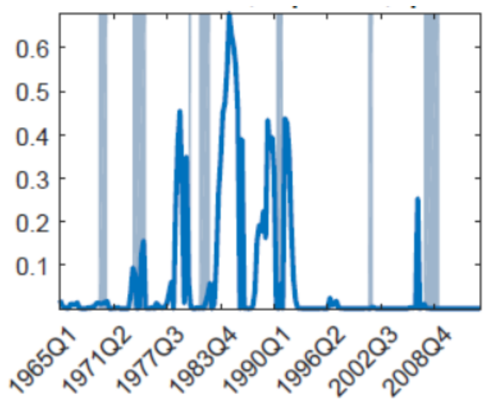

Figure 8: Smoothed probabilities for regime 8

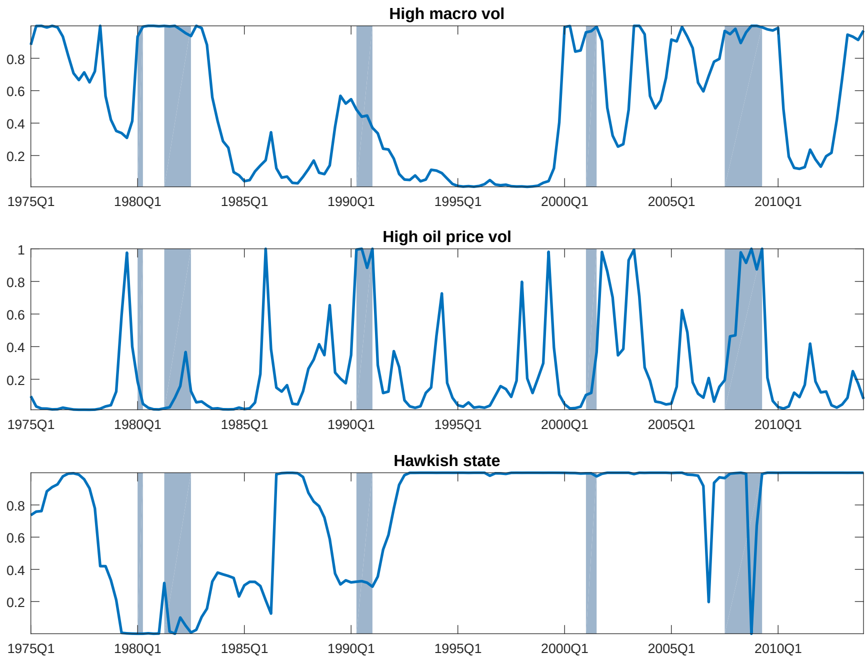

Figure 9: Smoothed state probabilities (starting in 1975Q1)

Note: The top row presents the smoothed probabilities for being in the high macroeconomic volatility regime. The second row presents the smoothed probabilities for being in the high oil price volatility regime. The bottom row presents the smoothed probabilities for being in the high monetary policy response regime.

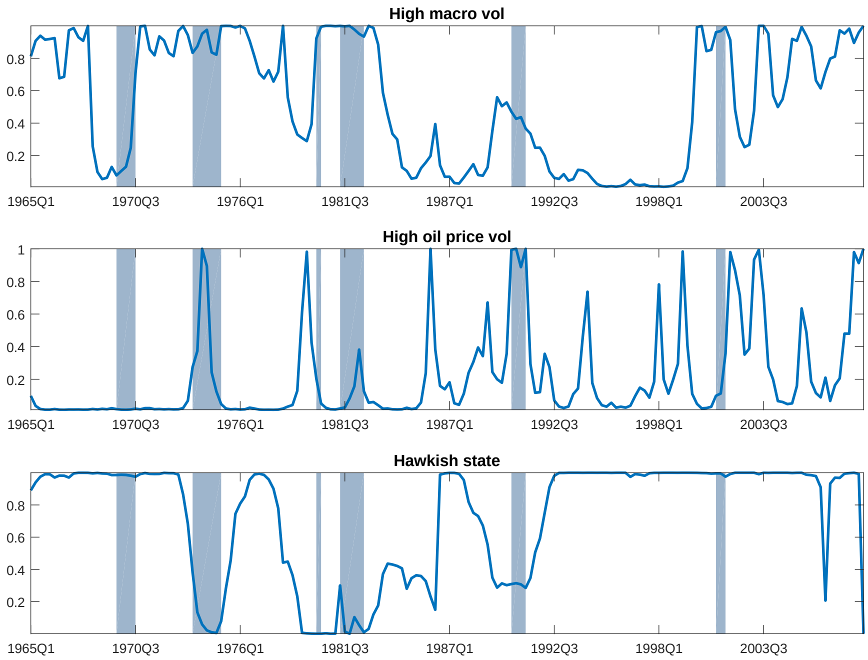

Figure 10: Smoothed state probabilities (ending in 2008Q4)

Note: The top row presents the smoothed probabilities for being in the high macroeconomic volatility regime. The second row presents the smoothed probabilities for being in the high oil price volatility regime. The bottom row presents the smoothed probabilities for being in the high monetary policy response regime.